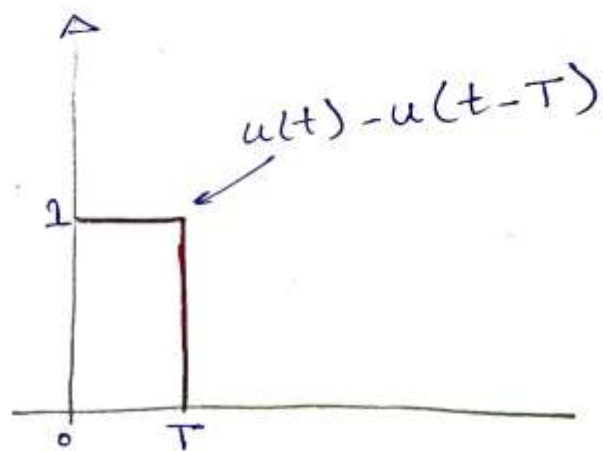
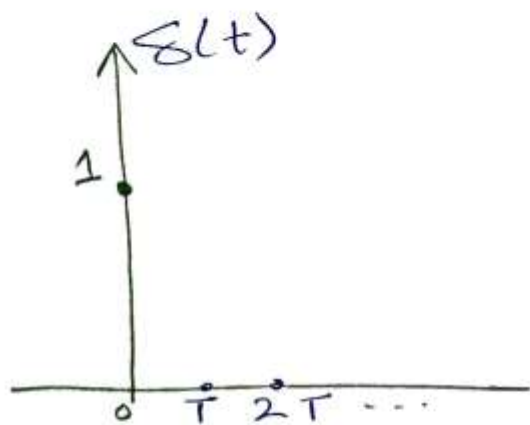
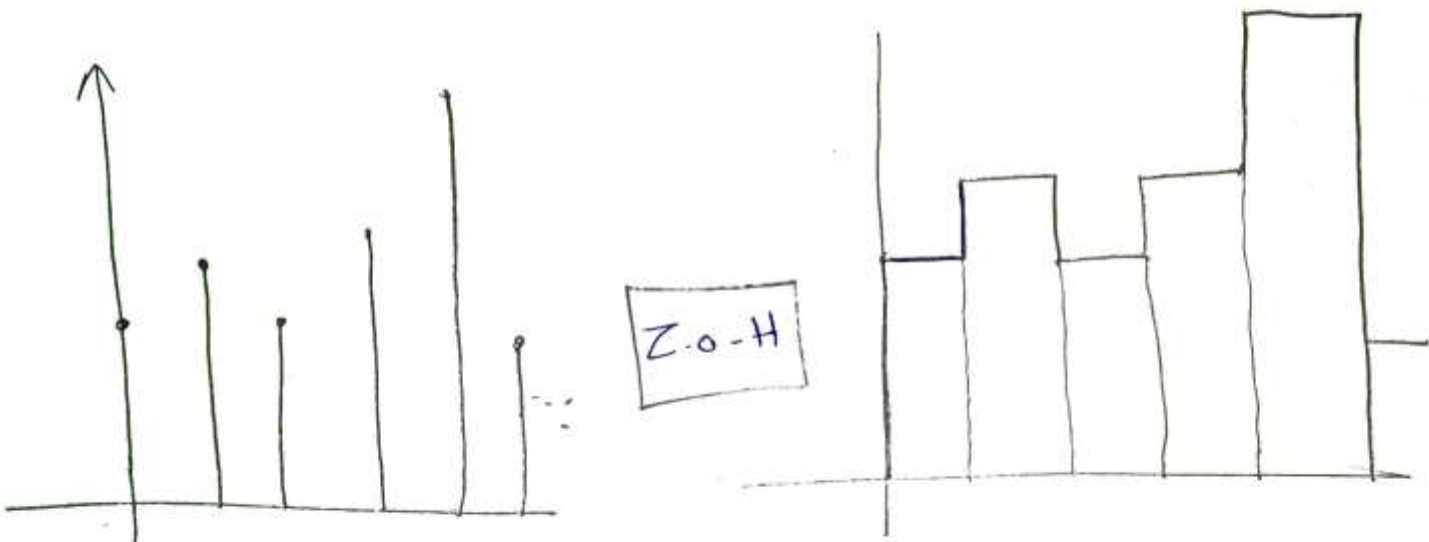


Digital Control

Lec 2

The T.F. of Z.o.H



$$\text{T.F. For Z.o.H} \Rightarrow G_{\text{Z.o.H}}(s) = G_h(s) = \frac{O/P(s)}{I/P(s)}$$

L.T

$$= \frac{\mathcal{L} \cdot (u(t) - u(t-T))}{\mathcal{L} \cdot (\delta(t))}$$

$$= \frac{\frac{1}{s} - \frac{\bar{e}^{-Ts}}{s}}{1} \cdot \frac{1}{s} (1 - \bar{e}^{-Ts})$$

Z.O.H = zero order hold

$$G(s)_{Z.O.H} = \frac{1 - \bar{e}^{-Ts}}{s}$$

~~Z.T~~ Z.T $G_h(s) \cdot G_1(s)$

$$Z \left[\frac{1 - \bar{e}^{-Ts}}{s} \cdot G_1(s) \right] = (1 - \bar{z}^{-1}) \cdot Z \left[\frac{G_1(s)}{s} \right]$$

* The zero order hold gain = 1

$$Z \cdot [G_h(s)] = Z \left[\frac{1 - \bar{e}^{-Ts}}{s} \right]$$

$$= (1 - \bar{z}^{-1}) \cdot Z \left[\frac{1}{s} \right]$$

$$= (1 - \bar{z}^{-1}) \left(\frac{z}{z-1} \right) = \frac{z-1}{z} * \frac{z}{z-1}$$

$$= 1$$

Pulse T.F

→ The discrete or digital T.F is called Pulse T.F.

$$\text{Pulse T.F} = \frac{C^*(s)}{R^*(s)} = \frac{C(z)}{R(z)}$$

① Pulse T.F from difference equations:-

For ex:

$$y(k-1) + 2y(k) + 3y(k-2) = r(k) \quad \downarrow \text{Z-T}$$

$$\text{Find Pulse T.F} = \frac{Y(z)}{R(z)}$$

$$z^{-1}Y(z) + 2Y(z) + 3z^{-2}Y(z) = R(z)$$

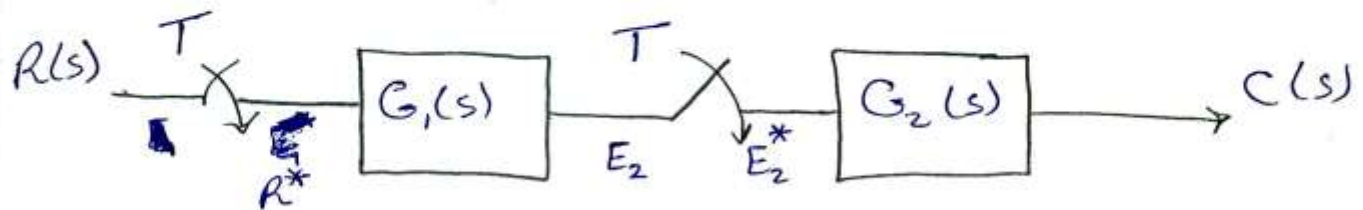
$$(3z^{-2} + z^{-1} + 2)Y(z) = R(z)$$

$$\frac{Y(z)}{R(z)} = \frac{1}{3z^{-2} + z^{-1} + 2} = \frac{z^2}{2z^2 + z + 3}$$

$$\frac{Y(z)}{R(z)} = \frac{z^2}{2z^2 + z + 3}$$

2] Pulse T.F from block diagram:-

a) open loop system:-



منه هتبقى دخل وخرج ال (Sampler) في E_2^* و E_2

$$C(s) = G_2(s) E_2^* \quad (1) \quad \text{~~(s)}~~$$

$$E_2 = G_1(s) R^* \quad (2)$$

← نقوم بعمل (staring) للمعادلات

$$C^*(s) = G_2^* E_2^* \quad (3)$$

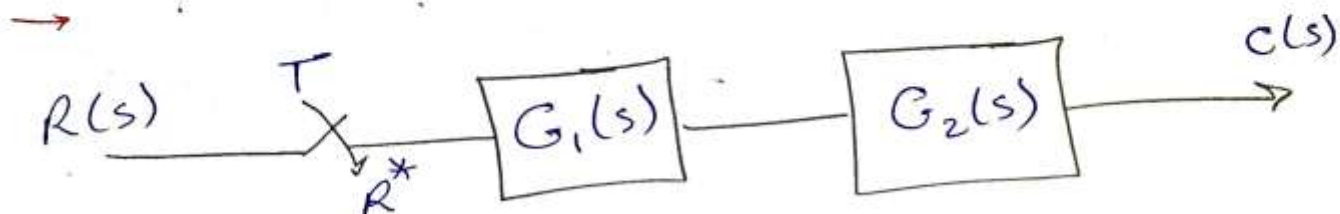
$$E_2^* = G_1^*(s) R^* \quad (4)$$

From (4) in (3)

$$C^*(s) = G_1^*(s) \cdot G_2^*(s) \cdot R^*$$

$$\frac{C^*(s)}{R^*(s)} = G_1^*(s) \cdot G_2^*(s) \quad \leftarrow \text{Pulse TF}$$

$$\boxed{\frac{C(z)}{R(z)} = G_1(z) \cdot G_2(z)} \leftarrow \text{Pulse T.F}$$



$$C(s) = G_1(s) \cdot G_2(s) \cdot R^*(s)$$

↓ sampling

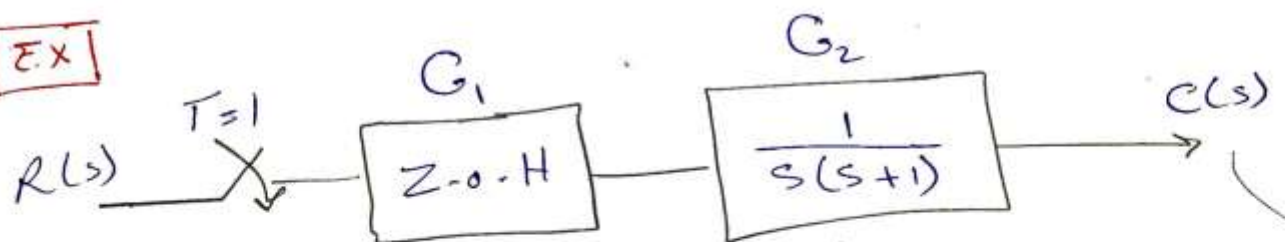
$$C^*(s) = G_1^*(s) \cdot G_2^*(s) \cdot R^*(s)$$

$$\boxed{\frac{C^*(s)}{R^*(s)} = G_1^*(s) \cdot G_2^*(s)}$$

$$\boxed{\frac{C(z)}{R(z)} = G_1(z) \cdot G_2(z)}$$

↖ Pulse T.F
↙

EX



1) Find T.F = $\frac{C(z)}{R(z)}$

2) Find the unit step response

$$\frac{C(z)}{R(z)} = \overline{G_1 G_2(z)}$$

$$= \mathcal{Z} \left(\frac{1 - e^{-Ts}}{s^2(s+1)} \right) = (1 - z^{-1}) \mathcal{Z} \left[\frac{1}{s^2(s+1)} \right]$$

$$= \frac{z-1}{z} \cdot \mathcal{Z} \left[\frac{A_1}{s^2} + \frac{A_2}{s} + \frac{A_3}{s+1} \right]$$

$$A_1 = 1, \quad A_3 = 1$$

$$s=1 \Rightarrow \frac{1}{2} = A_1 + A_2 + \frac{A_3}{2} \Rightarrow A_2 = -1$$

$$\frac{C(z)}{R(z)} = \left(\frac{z-1}{z} \right) \cdot \mathcal{Z} \left[\frac{1}{s^2} - \frac{1}{s} + \frac{1}{s+1} \right]$$

\uparrow t \uparrow 1 \uparrow e^{-t}

$$= \left(\frac{z-1}{z} \right) \left[\frac{Tz}{(z-1)^2} - \frac{z}{z-1} + \frac{z}{z - e^{-T}} \right]$$

$$= \left(\frac{z-1}{z} \right) \left[\frac{1}{(z-1)^2} - \frac{1}{z-1} + \frac{1}{z - e^{-1}} \right]$$

$$= \frac{1}{z-1} - 1 + \frac{z-1}{z-\bar{e}^1}$$

$$\bar{e}^1 \approx 0.368$$

$$\frac{C(z)}{R(z)} = \frac{(z-0.368) - (z-1)(z-0.368) + (z-1)^2}{(z-1)(z-0.368)}$$

$$\boxed{\frac{C(z)}{R(z)} = \frac{0.368z + 0.264}{(z-1)(z-0.368)}}$$

$b \rightarrow$ unit step response

$$r(t) = 1 \Rightarrow R(z) = \frac{z}{z-1}$$

$$C(z) = \left[\frac{0.368z + 0.264}{(z-1)(z-0.368)} \right] \cdot R(z)$$

$$= \frac{0.368z + 0.264}{(z-1)(z-0.368)} * \frac{z}{z-1}$$

$$C(z) = \frac{(0.368z + 0.264)z}{(z-1)^2(z-0.368)} \quad \Downarrow \quad z^{-1} \cdot T$$

(7)

$$c(z) = z \left[\frac{0.368z + 0.264}{(z-1)^2 (z-0.368)} \right]$$

⇓ using P.F

$$= z \left[\frac{A_1}{(z-1)^2} + \frac{A_2}{z-1} + \frac{A_3}{z-0.368} \right]$$

$$A_1 = 1, A_2 = -1, A_3 = 1$$

$$c(z) = z \left[\frac{1}{(z-1)^2} - \frac{1}{z-1} + \frac{1}{z-0.368} \right]$$

$$= \frac{z}{(z-1)^2} - \frac{z}{z-1} + \frac{z}{z-0.368}$$

⇓ $z^{-1} \cdot T$

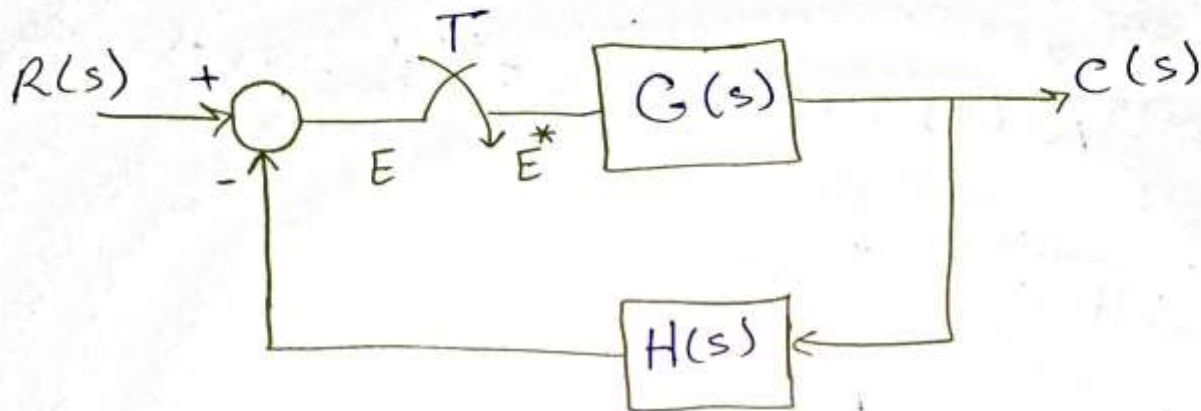
$$c(k) = k - u(k) + (0.368)^k$$

↳ unit step response

8

b) closed loop system

hint: From ways to find
Pulse T.F From block
diagram



(1) تسمية لدخل وخرج ال (Sampler).

(2) نحسب معادله الخرج الرئيس $C(s)$ بالافتراض لمعادلات

ال i/p s الخاصة بال (Sampler).

(3) قبل عمل (sampling) اذا تواجدت تقويضات نقوم بعملها.

(4) نعدل عملية (sampling) للمعادلات.

$$C(s) = G(s) E^*(s) \quad \rightarrow (1)$$

$$E(s) = R(s) - H(s) \cdot C(s)$$

$$= R(s) - G(s)H(s)E^*(s)$$

$$E(s) = R(s) - G(s)H(s) \cdot E^*(s) \quad \rightarrow (2)$$

Starting For ①, ②

$$C^*(s) = G^*(s) \cdot E^*(s) \rightarrow (3)$$

$$E^*(s) = R^*(s) - \overline{GH(s)}^* \cdot E^*(s) \rightarrow (4)$$

$$(1 + \overline{GH(s)}^*) E^*(s) = R^*(s)$$

$$E^*(s) = \frac{R^*(s)}{1 + \overline{GH(s)}^*} \rightarrow (5)$$

From (5) in (3)

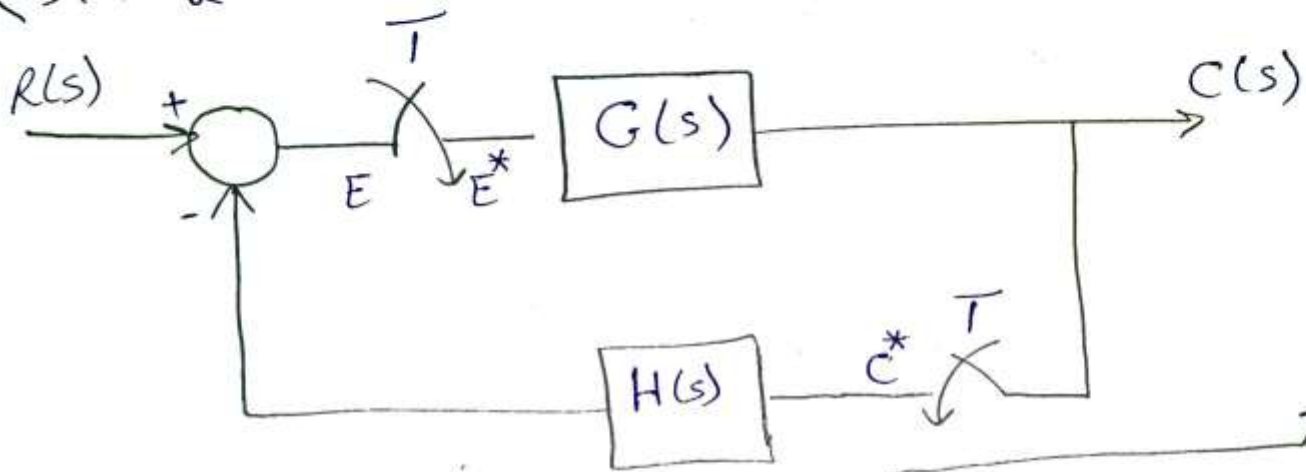
$$C^*(s) = \frac{G^*(s) \cdot R^*(s)}{1 + \overline{GH(s)}^*}$$

$$\frac{C^*(s)}{R^*(s)} = \frac{G^*(s)}{1 + \overline{GH(s)}^*}$$

$$\frac{C(z)}{R(z)} = \frac{G(z)}{1 + \overline{GH(z)}}$$

Pulse T.F

← إذا قمت بعمل (staring) قبل التعويض هتش هتخرف
 تحسب ال (Pulse T.F). (لكن فيه حالات بنعوض فيها بعد ال
 (staring



$$C(s) = G(s) \cdot E^*(s) \longrightarrow \boxed{C^*(s) = G^*(s) \cdot E^*(s)} \quad \text{①}$$

$$E = R - H(s) \cdot C^*(s) \longrightarrow E^* = R^* - H^* C^*(s)$$

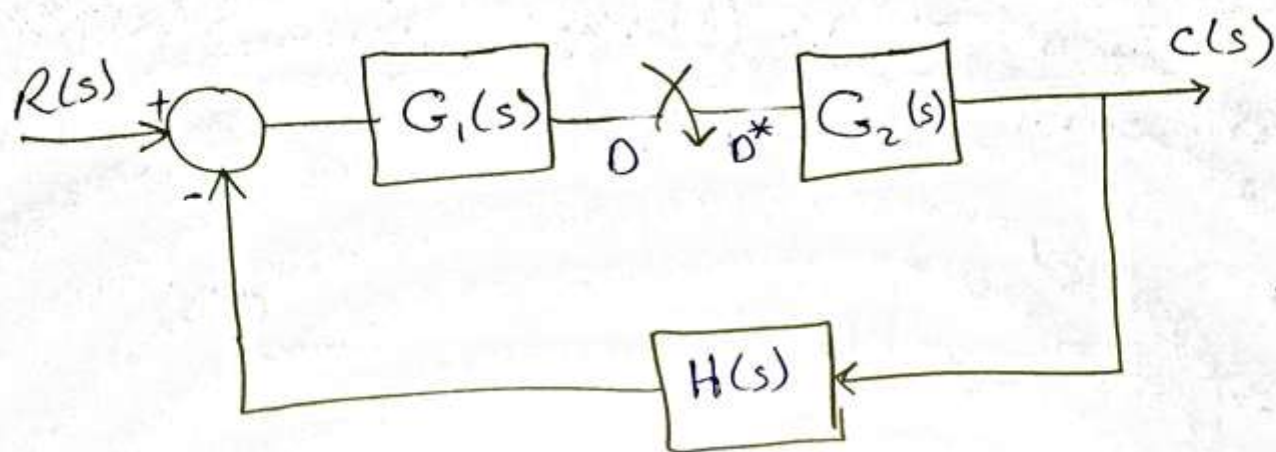
$$E^* = R^* - G^*(s) H^*(s) E^*(s)$$

$$(1 + G^*(s) H^*(s)) E^* = R^*$$

$$\boxed{E^* = \frac{R^*}{1 + G^*(s) H^*(s)}} \longrightarrow \text{③}$$

From ③ in ①

$$\boxed{\frac{C^*}{R^*} = \frac{G^*(s)}{1 + G^*(s) H^*(s)}}$$



$$C(s) = G_2(s) \cdot D^*(s) \rightarrow (1)$$

$$D(s) = G_1(s) [R(s) - C(s) - H(s) \cdot D^*(s)]$$

$$D(s) = G_1 R(s) - G_1 G_2 H(s) D^*(s) \rightarrow (2)$$

staring

$$C^*(s) = G_2^*(s) \cdot D^*(s) \rightarrow (3)$$

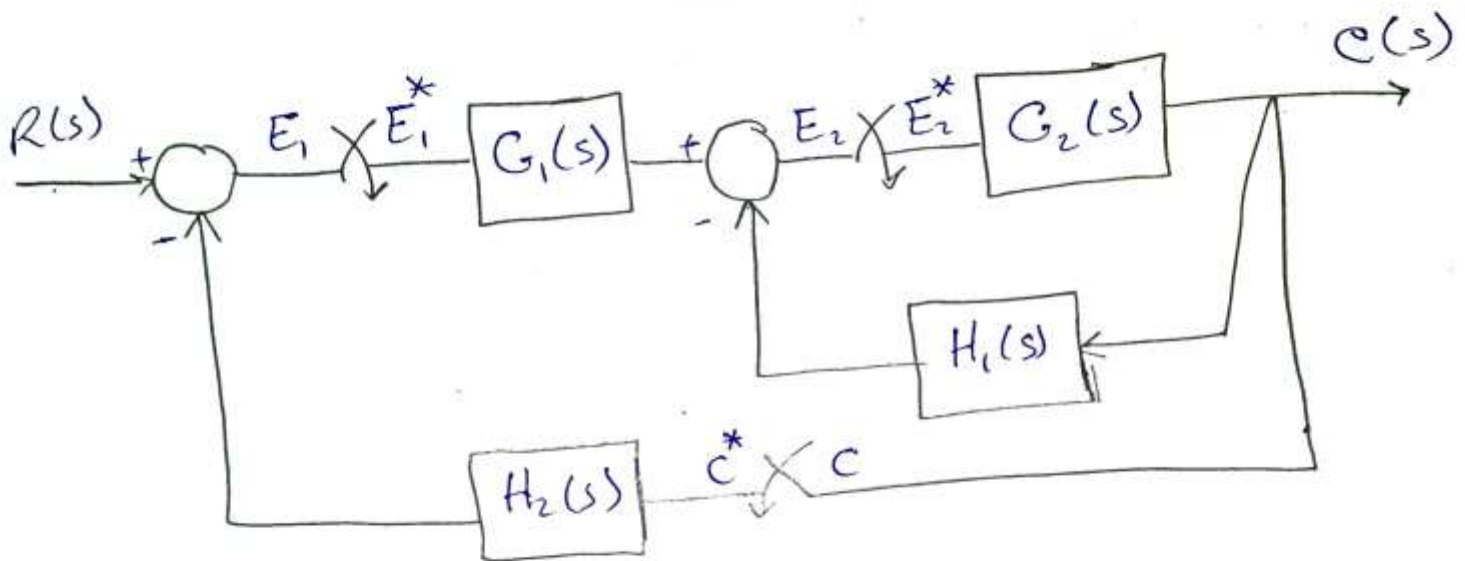
$$D^*(s) = \overline{G_1^*} R^*(s) - \overline{G_1 G_2 H^*} D^*(s) \rightarrow (4)$$

↳ we can't find impulse T.F

$$\cancel{D^*(s)} + C^*(s) = \frac{G_2^* \cdot \overline{G_1^*} R^*(s)}{1 + \overline{G_1 G_2 H^*}}$$

$$C(z) = \frac{G_2(z) \cdot \overline{G_1 R(z)}}{1 + \overline{G_1 G_2 H(z)}}$$

↳ system o/p



← معكته تتحسب بمجرد النظر

$$\frac{C(z)}{R(z)} = \frac{G_1(z) \cdot G_2(z)}{1 + \overline{G_2 H_1(z)} + \overline{G_1(z) \cdot G_2(z) H_2(z)}}$$

↓
إدخال (Feedback) على مخرج الرسم .

← لكننا مطالب في الامتحان بالخطوات .

$$C(s) = G_2(s) \cdot E_2^*(s) \rightarrow (1)$$

$$E_2(s) = G_1(s) E_1^* - G_2 H_1 E_2^* \rightarrow (2)$$

$$E_1(s) = R(s) - H_2(s) C^*(s) \rightarrow (3)$$

staring

$$C^*(s) = G_2^*(s) \cdot E_2^*(s) \rightarrow a$$

$$E_2^*(s) = G_1^*(s) E_1^* - \overline{G_2 H_1}^* E_2^* \rightarrow b$$

$$E_1^*(s) = R^*(s) - H_2(s) G_2^*(s) \cdot E_2^*(s) \rightarrow c$$

From c in b

$$\begin{aligned} E_2^* &= G_1^* (R^* - H_2^* G_2^* E_2^*) - \overline{G_2 H_1}^* E_2^* \\ &= G_1^* R^* - G_1^* G_2^* H_2^* E_2^* - \overline{G_2 H_1}^* E_2^* \end{aligned}$$

$$(1 + \overline{G_2 H_1}^* + G_1^* G_2^* H_2^*) E_2^* = G_1^* R^*(s)$$

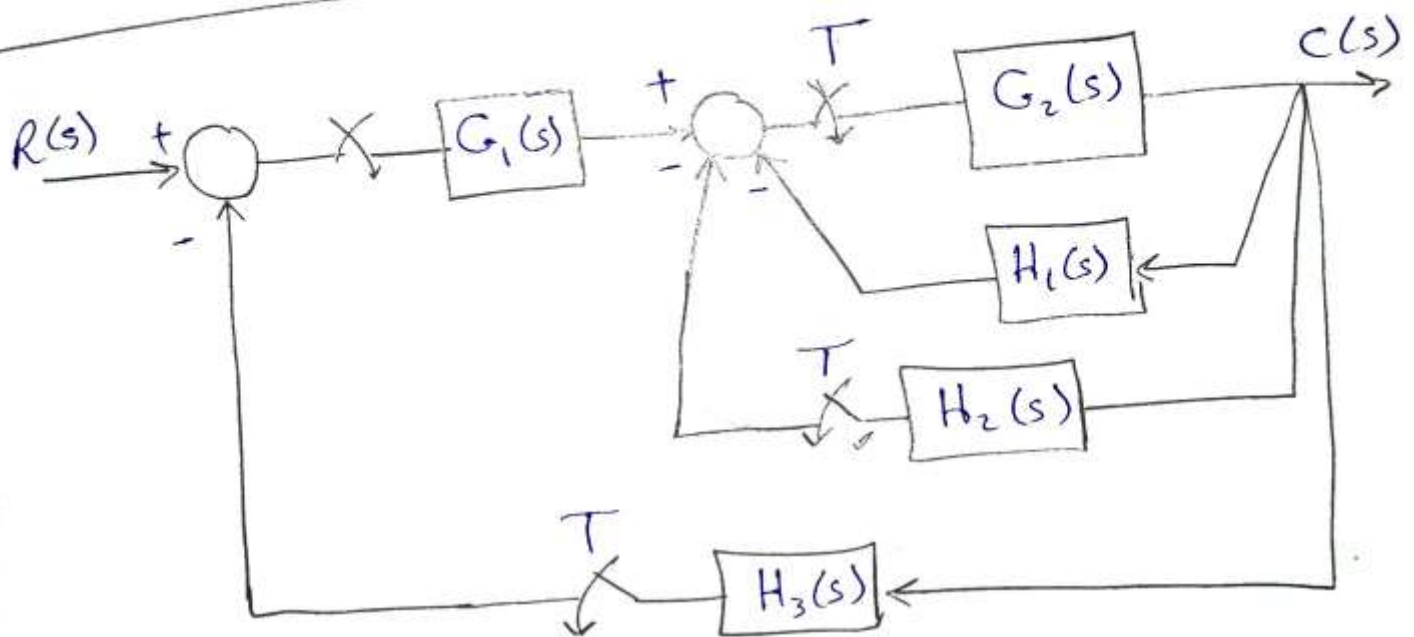
$$E_2^* = \frac{G_1^* R^*(s)}{1 + \overline{G_2 H_1}^* + G_1^* G_2^* H_2^*}$$

$\rightarrow d$

From (d) in (a)

$$\frac{C^*(s)}{R^*(s)} = \frac{G_1^*(s) \cdot G_2^*(s)}{1 + \overline{G_2 H_1}^* + G_1^* G_2^* H_2^*}$$

$$\frac{C(z)}{R(z)} = \frac{G_1(z) \cdot G_2(z)}{1 + \overline{G_2 H_1}(z) + G_1(z) \cdot G_2(z) H_2(z)}$$



← بالنظر

$$\frac{C(z)}{R(z)} = \frac{G_1(z) \cdot G_2(z)}{1 + \overline{G_2 H_1}(z) + \overline{G_2 H_2}(z) + G_1(z) \cdot G_2 H_3(z)}$$

لو مكان ال (Sampler) في الاتجاه الآخر هيفرق
لأنه هينصل ما بين آخر حدين في المقام.